P8–6.–The following differential equations describe the motion of a body in orbit about two much heavier bodies. An example would be an Apollo capsule in an earth-moon orbit. The coordinate system is a little tricky. The three bodies determine a plane in space and a two-dimensional cartesian coordinate in this plane. The origin is at the center of mass of the two heavy bodies, the x axis is the line through these two bodies, and the distance between them is taken as the unit. Thus, if μ is the ratio of the mass of the moon to that of the earth, then the moon and the earth are located at coordinates $(1-\mu, 0)$ and $(-\mu, 0)$, respectively, and the coordinate system moves as the moon rotates about the earth. The third body, the Apollo, is assumed to have a mass which is negligible compared to the other two, and its position as a function of time is $(x(t), y(t))$. The equations are derived from Newton's law of motion and the inverse square law of gravitation. The first derivatives in the equation come from the rotating coordinate system and from a frictional term, which is assumed to be proportional to velocity with proportionality constant f:

$$x'' = 2y' + x - \frac{\tilde{\mu}(x+\mu)}{r_1^3} - \frac{\mu(x-\tilde{\mu})}{r_2^3} - fx',$$

$$y'' = -2x' + y - \frac{\tilde{\mu}y}{r_1^3} - \frac{\mu y}{r_2^3} - fy',$$

with

$$\mu = \frac{1}{82.45}, \quad \tilde{\mu} = 1 - \mu, \quad r_1^2 = (x+\mu)^2 + y^2, \quad r_2^2 = (x-\tilde{\mu})^2 + y^2.$$

Although a great deal is known about these equations, it is not possible to find closed-form solutions. One interesting class of problems involves the study of periodic solutions in the absence of friction. It is known that the initial conditions

$$x(0) = 1.2, \quad x'(0) = 0, \quad y(0) = 0, \quad y'(0) = -1.04935751$$

lead to a solution which is periodic with period $T = 6.19216933$, when $f = 0$. This means that the Apollo starts on the far side of the moon with an altitude of about 0.2 times the earth-moon distance and a certain initial velocity. The resulting orbit brings the Apollo in close to the earth, out in a big loop on the opposite side of the earth from the moon, back in close to the earth again, and finally back to its original velocity on the far side of the moon.

(a) Use SDRIV2 to compute the solution with the given initial conditions. Verify that the solution is periodic with the given period. How close does the Apollo come to the *surface* of the earth in this orbit? In the equation, distances are measured from the centers of the earth and moon. Assume that the moon is 238,000 miles from the earth and that the earth is a sphere with radius of 4000 miles. Note that the origin of the coordinate system is within this sphere but not at its center.

(b) When $f = 1$ with the same initial conditions as in (a), integrate from $0 \le t \le 5$. Plot the phase plane of the solution. That is, plot $x(t)$ versus $y(t)$. In this case, the Apollo is captured by the earth and eventually "crashes."

(c) When $f = 0.1$ repeat the computations of (b). By looking at the phase plane, can you guess what is happening? This will be easier to understand if you perform the integration for a longer time, say to $t = 8$.